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***Abstract*—In this work, we proposed a tractable mathematical framework to analyze the coverage probability in dynamic downlink cellular networks taking into account the queue dynamics. In particular, the static properties of the physical layer network are studied by stochastic geometry and dynamic properties of queue evolution are studied with discrete time Markov chain. We also analyze the stable region by giving explicit upper and lower bounds on the dynamic coverage probability.**

***Index Terms*—stochastic geometry, downlink cellular network, instantaneous SINR, DTMC, stability.**

I. Introduction

A. Background and related work

The stochastic geometry approach has recently got much attention in particular for quantifying the aggregate interference in the wireless network. The introduction of point process theory turned out to be a great tool for modeling and analyzing the performance of wireless networks in different scenarios [1]–[3]. The first works focused on full-load networks, i.e. transmitters never have empty buffers and then transmit all the time, and hence the studies investigated the impact of a random topology on the coverage probability of the typical user.

However, real systems are subjected to temporal traffic variation and sources generate packets according to some stochastic process which should be stored in a queue before being transmitted. The introduction of queueing theory in a stochastic geometry approach allows to assess important network performance measures such as the average delay or stability to cite a few. The analysis remains however challenging due to the complex interaction between the packet arrival rate process and the service rate depending on the coverage probability that in turn depends on the interference in the networks and all the queue states of transmitters. A first attempt combining stochastic geometry and queueing theory has been granted in [4] by considering a double-stochastic network, i.e. space-time Poisson call arrivals. Authors succeeded to derive network performance measures averaging over space and time. However, the interaction between the queues at different BSs are either ignored or only analyzed by approximations.

The works in [5]–[7] pushed further the analysis of the interaction between the queue dynamics and the topology of the network. A traffic-aware spatio-temporal model for IoT devices supported by cellular uplink connectivity has been developed in [5]. A quite complete transmission scheme, i.e. backoff and transmission power, has been proposed using Markov chains whose evolution depends on the queue state of the devices. Thanks to this model, authors studied the tradeoff between the scalability of the network, i.e. supporting as much as possible a high number of devices, and its stability, i.e. queues are not diverging. A marked Poisson process is modeled to catch arrival packets and delays in heterogeneous cellular network [6]. Similarly, [7] assumed that the traffic is generated at random spatial regions, rather than modeling the flow at each independent user. In [8], a novel spatio-temporal mathematical framework is provided to analyze the preamble transmission success probability of a cellular IoT network, where the number of accumulated packets in the queues is approximated by a Poisson distribution. However, the theoretical findings are not validated by simulations.

Further, the primal consideration in queueing systems is about stability. For an isolated system, even with random arrival and departure process, the stable region requires that the service rate is larger than the arrival rate. However, the sufficient conditions are more complicated in large-scale queueing networks since the service rate depends on the state of all the transmitters in the network. Sufficient and necessary conditions for system stability have been studied in [9], meta-stability in [10]. Particularly, [11] characterizes the SIR variation due to the traffic conditions, and its impact on system stability. However, the stable region is characterized by assuming a dominant and a modified system to avoid the problem of interacting queues. Thus, a simple but general framework to describe the dynamic interaction effects of coverage probability remains to propose.

B. Approach and contributions

This work proposes a tractable mathematical model to analyze the coverage probability in a downlink cellular network, in which queue dynamics are taken into consideration. We develop a comprehensive approach to handle the interaction between the coverage probability and the queueing state evolution using discrete time Markov chain (DTMC). A simple model is considered, but contrarily to the state of the art [9]–[11], closed-form expressions are given that make the bridge between the coverage probability and the fraction of

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active base stations (BS) under conditional stable state. We also characterize the explicit upper and lower bounds on the dynamic coverage probability.

The rest of paper is organized as follows. Section II presents the system model and the assumptions. The outline of the analytical framework is established in Section III. Section IV provides the simulation results and conclusions are drawn in Section V.

II. SYSTEM MODEL

A. Network topology

The downlink cellular-based network model consists of a single-tier base stations (BSs) in $\mathbb{R}^2$ following an independent homogeneous Poisson point process (PPP) Φ, with intensity λ. User density is such that every BS has at least one user associated with it . Besides, each user is associated to the closest BS. A single user equipment (UE), randomly chosen, is considered as a typical UE and being located at origin $(0, 0)$ for the ease of the analysis. Moreover, all BSs are assumed to transmit with a constant normalized power in the same spectrum.

A single queue but multiple servers are considered per BS. The study focuses on a given resource block per user and an independently and identically distributed (i.i.d.) flat fading channel across time and space is considered [11].

B. Traffic model

We use a discrete time queueing system to model the random traffic arrival and departure processes. The packet arrival is modeled by an independent Bernoulli process with rate $\xi \in [0, 1]$ and the rate of the departure random process of the queue depends on the instantaneous signal-to-noise ratio (SINR). If the received SINR exceeds a predefined threshold θ, the packet is transmitted successfully and it can be removed from the queue. Otherwise, the transmission fails and the packet remains in the buffer waiting for re-transmission in the next time slot. Moreover, whether the transmission of the typical BS is successful or not, the queue evolution depends on the realization of the PPP, the random access, the fading and the arrival traffic rates, and the dynamic aggregate interference.

The realization of the point process Φ is conditioned on a full time activity of the BS at position x_0. The relevant probability measure over the PPP is then the reduced Palm probability, denoted as $\mathbb{P}^{x_0}$. Correspondingly, the expectation is taken with respect to the measure $\mathbb{P}^{x_0}$.

Furthermore, we define Φ_t to be the set of BSs that are transmitting in the time slot $t \in \mathbb{N}$. Obviously, a transmitter with a reasonable packet input rate and experiencing a favorable network performance (i.e. high successful transmission probability) is able to clear the backlogged packets in the buffer. Otherwise, the backlogged packets in the buffer will rapidly grow up.

C. Signal-to-interference ratio

The aggregate interference of the typical UE at time slot t can be written as

$$I_t = \sum_{x \in \Phi \setminus x_0} h_{x,t} \left\| x \right\|^{-\alpha} \mathbb{1}(x \in \Phi_t) \tag{1}$$

where $\|x\|$ is the distance between the interfering BS at x and the typical user, $h_{x,t}$ is the exponential channel gain between the typical UE and the interfering BS at position x and time t, with mean 1, α is the path loss exponent, and $\mathbb{1}(\cdot)$ is the indicator function.

Hence, the received SINR at time slot t experienced by the typical UE is

$$\gamma_t = \frac{h_{x_0,t} \left\| x_0 \right\|^{-\alpha}}{\sigma^2 + \sum\limits_{x \in \Phi \setminus x_0} h_{x,t} \left\| x \right\|^{-\alpha} \mathbb{1}(x \in \Phi_t)} \tag{2}$$

where σ^2 denotes the power of additive white Gaussian noise and $h_{x_0,t}$ is the exponential channel gain between the typical UE and its tagged BS. Moreover, we note q_t the quantity

$$q_t = \mathbb{E}_{\mathbb{1}(x \in \Phi_t)}(\mathbb{1}(x \in \Phi_t) = 1) \tag{3}$$

which can be seen as (i) the fraction of active interfering BS at time slot t; (ii) the probability that a randomly chosen BS is active at time slot t.

III. PERFORMANCE ANALYSIS

Our main results are stated in Theorems 1 and 2 in this section. The traffic analysis and its relation with the coverage probability are performed thanks to a DTMC. Finally, upper and lower bounds on the dynamic coverage probability are derived.

A. Coverage probability

Considering that the typical UE receives data at time slot t, i.e. its associated BS in x_0 is always active, the coverage probability is defined as [9]

$$p_t(\theta, \lambda, \alpha, \xi) \triangleq \mathbb{P}^{x_0}[\gamma_t > \theta] \tag{4}$$

Theorem 1. *The conditional coverage probability at time t is*

$$p_t(\theta, \lambda, \alpha, \xi) = 2\pi\lambda \int_0^{\infty} e^{-\sigma^2 \theta r^\alpha} e^{-\pi\lambda r^2 (1 + q_t \rho(\alpha, \theta))} r \mathrm{d}r$$

where $\rho(\alpha, \theta) = \int_1^\infty [1 + u^{\frac{\alpha}{2}} \theta^{-1}]^{-1} du.$

Proof. See Appendix A. □

Theorem 1 quantifies how the coverage probability behaves at a given time slot and depends on the traffic. It illustrates that the state of the queues are affecting the coverage via the parameter q_t. As q_t depends on the time t, the coverage probability in Theorem 1 is time depending. In the particular case of the interference-limited network, Theorem 1 takes the following form.

Corollary 1. *In an interference-limited network, i.e.* $\sigma^2 \to 0$, *we have*

$$p_t(\theta,\lambda,\alpha,\xi) = \left[1 + \int_1^{\infty} \frac{q_t}{1+u^{\frac{\alpha}{2}}\theta^{-1}} du\right]^{-1} \quad (5)$$

and for path loss exponent $\alpha = 4$, *we have*

$$p_t(\theta,\lambda,4,\xi) = \left[1 + q_t\sqrt{\theta}\tan^{-1}(\sqrt{\theta})\right]^{-1} \quad (6)$$

Either in Theorem 1 or Corollary 1, the coverage probability depends on the BS activity probability, i.e. q_t, which is related to the queue dynamic.

B. Queueing analysis

Let $\widetilde{\Phi}$ be the limit of Φ_t as $t \to \infty$, which represents the point process in the stationary regime, assuming it exists. Let p be the coverage probability of typical user at a stable state and $q = \mathbb{E}_{\mathbb{1}(x\in\widetilde{\Phi})}(\mathbb{1}(x \in \widetilde{\Phi}) = 1)$. Considering $q_{t\to\infty} = q$, and replacing q_t by q in Theorem 1, we have

$$\begin{aligned} p(\theta,\lambda,\alpha,\xi) &= \lim_{t\to\infty} p_t(\theta,\lambda,\alpha,\xi) \\ &= 2\pi\lambda \int_0^{\infty} e^{-\sigma^2\theta r^{\alpha}} e^{-\pi\lambda r^2(1+q\rho(\alpha,\theta))} r\mathrm{d}r \quad (7) \end{aligned}$$

The coverage probability given in (7) does not depend on t, contrary to the coverage probability given in Theorem 1. This coverage probability is then a stable coverage probability.

Theorem 2. *Under prescribed system assumption, let* ξ *and* p *be the arrival and departure rates, the active probability at a randomly chosen BS is*

$$q = \begin{cases} \xi/p, & \text{if } p > \xi, \\ 1, & \text{if } p \le \xi. \end{cases} \quad (8)$$

Proof. See Appendix B. ☐

According to (7) and (8), the interdependence between q and p shows the relationship between the queue and the stochastic geometry in the analysis. According to the relative values of p and ξ, a randomly chosen BS has a probability of ξ/p to be active if its arrival rate is less than the departure rate, and is always active in the opposite case. The computation of the probability q is performed dynamically thanks to Algorithm 1.

Algorithm 1 Iteration algorithm for computation of p and q.

Initialize $q_1 \in (\xi, 1)$, $q_0 = 0$, $i = 0$, $\epsilon \ll 1$
while $|q_{i+1} - q_i| \ge \epsilon$ **do**
  $i \leftarrow i+1$, $q \leftarrow q_i$, $p \leftarrow p(\theta,\lambda,\alpha,\xi)$ (7)
  if $p > \xi$ **then**
    $q_{i+1} \leftarrow \xi/p$
  else
    $q_{i+1} \leftarrow 1$
    break
  end if
end while
Return $q \leftarrow q_{i+1}$ and $p \leftarrow p(\theta,\lambda,\alpha,\xi)$

It is important to note when $p \le \xi$, $q = 1$ and all the buffers length grow up to infinity, i.e. the DTMC are not stable. The stable coverage probability (7) can however be defined but at the cost of infinite queue lengths or dropped packets.

C. Upper and lower bounds

The solution of the stability of the network in coverage and activity probabilities has been given in the previous section and can be numerically computed using Algorithm 1. Simulation results will show that this stability behaves between two extreme cases that are summarized in the next lemma.

Lemma 1. *Considering the depicted downlink cellular network, the coverage probability can be bounded as follows*

$$p_{\mathrm{l}} \le p \le p_{\mathrm{u}} \quad (9)$$

where

$$p_{\mathrm{u}} = 2\pi\lambda \int_0^{\infty} \exp(-\sigma^2\theta r^{\alpha}) e^{-\pi\lambda r^2(1+\xi\rho(\alpha,\theta))} r\mathrm{d}r \quad (10)$$

and

$$p_{\mathrm{l}} = 2\pi\lambda \int_0^{\infty} \exp(-\sigma^2\theta r^{\alpha}) e^{-\pi\lambda r^2(1+\rho(\alpha,\theta))} r\mathrm{d}r \quad (11)$$

Proof. A favorable system is considered for the upper bound [9]. If the transmission of a packet fails, this packet is dropped instead of being re-transmitted. The interfering transmitter is then active with probability ξ, or we can say that the fraction of the active interfering transmitters remains constant in time. Substituting q_t in (5) by its minimum value $q_t = \xi$, the upper bound is obtained.

In the lower bound case, the highest interference situation is obtained when all BSs are always active [9]. This corresponds to $q_t = 1$, which also gives the lowest value of the function in (5). ☐

It has to be mentioned that the lower bound (11) is the coverage probability given in [1], and the upper bound (10) is the coverage probability given in [1] with a BS density thinned by a factor ξ. As with Theorem 2, the stability condition in Lemma 1 is ensured at a cost of infinite buffer lengths if $p_{\mathrm{l}} \le \xi$. Moreover, the bounds in Lemma 1 reduce to a much more simpler form when the network is considered as interference-limited.

Corollary 2. *In an interference-limited network, i.e.* $\sigma^2 \to 0$,

$$p_{\mathrm{u}} = [1 + \xi\rho(\alpha,\theta)]^{-1} \quad (12)$$

$$p_{\mathrm{l}} = [1 + \rho(\alpha,\theta)]^{-1} \quad (13)$$

IV. Numerical results and simulations

Fig. 1 plots the coverage probability expressed in Theorem 1 w.r.t. the threshold θ. Moreover, two initialization states are considered: the full load case, i.e. the lower-bound in Lemma 1, and the light traffic initialization case, i.e. the upper-bound in Lemma 1. Moreover, the time evolution of the coverage probability when the number of time slots increases

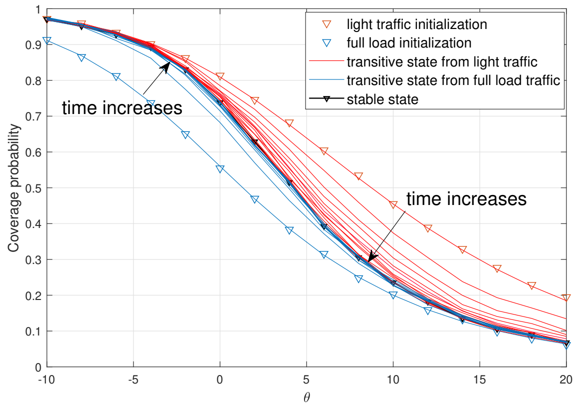


Figure 1. Comparison of dynamic coverage probability with two initialization, $\xi = 0.3$, $\sigma^2 = 0$, $\lambda = 0.25$ and $\alpha = 4$.

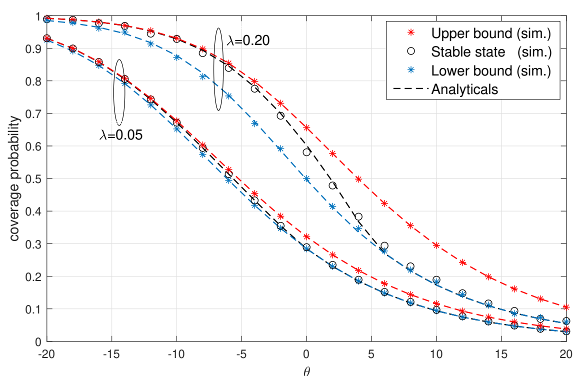


Figure 2. Comparison of Monte Carlo simulation and analytically iterative algorithm of coverage probability at stable state.

is illustrated thanks to the arrows in Fig. 1. Whatever the initialization state is, the coverage probability converges to the stable coverage probability, corresponding to the stable distribution of the DTMC when $p \geq \xi$.

Fig. 2 compares the analytical results in Theorem 2, evaluated with Algorithm 1, with the Monte Carlo simulations under two network densities, $\lambda = 0.05$ and $\lambda = 0.2$. The average arrival rate is set to $\xi = 0.3$ and $\sigma^2 = 0.1$. The results corroborate the good match between simulations and analytical expressions. Moreover, we observe that the region between upper and lower bounds reduces when λ decreases. Indeed, as the density becomes lower, the interference level at the typical user decreases also and hence the upper bound is close to the lower bound.

V. Conclusion

This work proposed a tractable mathematical model to analyze the coverage probability in a dynamic traffic randomly deployed downlink cellular network. The queue evolution at each transmitter has been handled with a DTMC and a Bernoulli distribution for packet arrival. The interaction between the coverage probability and the queue state evolution has been captured in closed-form. We also develop the boundaries of dynamic coverage probability. As further work, we intend to investigate the analysis when the traffic arrival rate is a function of the number of users in the network. Also, a multi-devices competition mechanism can be introduced, which means that in the give time slot the BS has more than one queue that needs to be served.

Appendix

A. Proof of Theorem 1

Given the typical UE received data at time slot t, its conditional SINR coverage probability is written as (to lighten the notation we remove the index t from the channel coefficients)

$$
\begin{aligned}
p_t &= \mathbb{P}^{x_0}(\gamma_t \geq \theta) \\
&= \mathbb{P}^{x_0}\left[\frac{h_{x_0}\|x_0\|^{-\alpha}}{\sigma^2 + \sum\limits_{x\in\Phi\backslash x_0} h_x\|x\|^{-\alpha}\mathbb{1}(x\in\Phi_t)} \geq \theta\right] \\
&= \int_0^\infty 2\pi\lambda r_0 e^{-\pi\lambda r_0^2}\exp(-\sigma^2\theta r_0^\alpha) \\
&\quad \times \mathbb{P}^{x_0}\left[\frac{h_{x_0}\|x_0\|^{-\alpha}}{\sigma^2 + \sum\limits_{x\in\Phi\backslash x_0} h_x\|x\|^{-\alpha}\mathbb{1}(x\in\Phi_t)} \geq \theta \middle| \|x_0\| = r_0\right]\mathrm{d}r_0 \\
&= \int_0^\infty 2\pi\lambda r_0 e^{-\pi\lambda r_0^2} e^{-\sigma^2\theta r_0^\alpha}\mathcal{L}_I(\theta r_0^\alpha)\mathrm{d}r_0 \qquad (14)
\end{aligned}
$$

where the Laplace transform (LT) of a random variable X in s is denoted as $\mathcal{L}_X(s)$.

Further, the LT $\mathcal{L}_I(s)$ in (14), with $s = \theta\|x_0\|^\alpha = \theta r_0^\alpha$, has the form:

$$
\begin{aligned}
\mathcal{L}_I(s) &= \mathbb{E}\left[\exp\left(-s\sum_{x\in\Phi\backslash x_0} h_x\|x\|^{-\alpha}\mathbb{1}(x\in\Phi_t)\right)\middle| r_0\right] \\
&= \mathbb{E}_{\{h_x\},\Phi}\left[\prod_{x\in\Phi\backslash x_0}\exp\Big(-s h_x\|x\|^{-\alpha}\mathbb{1}(x\in\Phi_t)\Big)\middle| r_0\right] \\
&\overset{a}{=} \mathbb{E}_\Phi\left[\prod_{x\in\Phi\backslash x_0}\mathbb{E}_{h_x}\left[\exp\Big(-s h_x\|x\|^{-\alpha}\mathbb{1}(x\in\Phi_t)\Big)\right]\middle| r_0\right] \\
&= \mathbb{E}_\Phi\left[\prod_{x\in\Phi\backslash x_0}\frac{1}{1+s\|x\|^{-\alpha}\mathbb{1}(x\in\Phi_t)}\middle| r_0\right] \\
&= \mathbb{E}_{\{\mathbb{1}(x\in\Phi_t)\}}\left(\mathbb{E}_\Phi\left[\prod_{x\in\Phi\backslash x_0}\frac{1}{1+s\|x\|^{-\alpha}\mathbb{1}(x\in\Phi_t)}\middle| \begin{matrix} r_0, \\ \mathbb{1}(x\in\Phi_t)\end{matrix}\right]\right) \\
&\overset{b}{=} \mathbb{E}_\Phi\left[\prod_{x\in\Phi\backslash x_0}\left(\frac{\mathbb{E}_{\mathbb{1}(x\in\Phi_t)}[\mathbb{1}(x\in\Phi_t)=1]}{1+s\|x\|^{-\alpha}\times 1}\right.\right. \\
&\qquad \left.\left. + \frac{\mathbb{E}_{\mathbb{1}(x\in\Phi_t)}[\mathbb{1}(x\in\Phi_t)=0]}{1+s\|x\|^{-\alpha}\times 0}\right)\middle| r_0\right]
\end{aligned}
$$

$$\overset{c}{=} \mathbb{E}_{\Phi}\left[\prod_{x\in\Phi\backslash x_0}\left(\frac{q_t}{1+s\left\|x\right\|^{-\alpha}}+1-q_t\right)\middle| r_0\right] \tag{15}$$

where (a) follows from the i.i.d. hypothesis of h_x and further independence from the point process Φ, (b) follows from the law of total expectation and using independence activity of BS [5, Assumption 2], and (c) follows from (3).

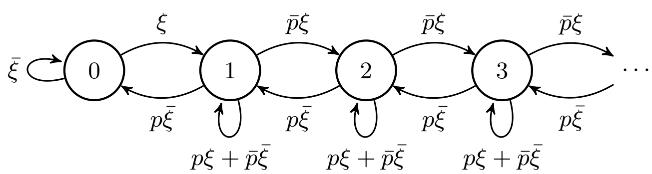
According to the PGFL of PPP and with $r = \|x\|$, we have:

$$\begin{aligned}&\mathcal{L}_I(\theta r_0^{\alpha})\\ &= \exp\left(-2\pi\lambda\int_{r_0}^{\infty}\left(1-\left(\frac{q_t}{1+\theta r_0^{\alpha} r^{-\alpha}}+1-q_t\right)\right)r\mathrm{d}r\right)\\ &\overset{a}{=} \exp\left(-\pi\lambda r_0^2\int_{1}^{\infty}\frac{q_t}{1+u^{\frac{\alpha}{2}}\theta^{-1}}\mathrm{d}u\right) \end{aligned}\tag{16}$$

where (a) is obtained by the change of variable $u = (\frac{r}{r_0})^2$.

B. Proof of Theorem 2

The number of packets in the queue of a randomly chosen BS is modeled as a birth and death process that can be represented with the following DTMC:



where $\bar{a} = 1 - a$ with $a \in \{p, \xi\}$ and each state is the number of packets being in the queue at a given time slot. When the stable state is achieved, the coverage probability of typical UE does not evolve with time as mention in (7). The packet departure at a given interfering BS is a Bernoulli process, which leads to a geometric inter-departure time. Thus, the transmission matrix of DTMC is a Geo/Geo/1 queueing model [12], and the transition matrix is

$$\mathbf{P} = \begin{bmatrix} \bar{\xi} & \xi & 0 & 0 & 0 & \cdots \\ p\bar{\xi} & \bar{p}\bar{\xi}+p\xi & \bar{p}\xi & 0 & 0 & \cdots \\ 0 & p\bar{\xi} & \bar{p}\bar{\xi}+p\xi & \bar{p}\xi & 0 & \cdots \\ 0 & 0 & p\bar{\xi} & \bar{p}\bar{\xi}+p\xi & \bar{p}\xi & \cdots \\ 0 & 0 & 0 & \ddots & \ddots & \ddots \end{bmatrix} \tag{17}$$

For stationary Markov chains, we have

$$\boldsymbol{\pi}\mathbf{P} = \boldsymbol{\pi},\ \boldsymbol{\pi}\boldsymbol{e} = 1 \tag{18}$$

where $\boldsymbol{\pi} = [\pi_0, \pi_1, \pi_2, \cdots, \pi_i, \cdots]$ is the row vector that contains the stable-state probabilities, in which π_i denotes the probability of being in state with i packets, and $\boldsymbol{e}$ is a column vector of ones with the proper length.

The solution of (18) is the solution of:

$$\begin{cases} \bar{\xi}\pi_0 + \bar{\xi}p\pi_1 = \pi_0 \\ \xi\pi_0 + (\bar{\xi}\bar{p} + \xi p)\pi_1 + \bar{\xi}p\pi_2 = \pi_1 \\ \xi\bar{p}\pi_1 + (\bar{\xi}\bar{p} + \xi p)\pi_2 + \bar{\xi}p\pi_3 = \pi_2 \\ \xi\bar{p}\pi_2 + (\bar{\xi}\bar{p} + \xi p)\pi_3 + \bar{\xi}p\pi_4 = \pi_3 \\ \qquad\qquad\vdots \end{cases} \tag{19}$$

Solving the system in (19) leads to:

$$\pi_i = R^i\frac{\pi_0}{\bar{p}},\ \text{where}\ R = \frac{\xi\bar{p}}{\bar{\xi}p},\ \forall\ i \in [1, +\infty) \tag{20}$$

By the law of total probability we should have

$$\sum_{i=0}^{\infty}\pi_i = 1 \tag{21}$$

it comes

$$\pi_0 + \frac{\pi_0}{\bar{p}}\sum_{i=1}^{\infty}R^i \overset{a}{=} \pi_0\left(1 + \frac{1}{\bar{p}}\times\frac{R}{1+R}\right) = 1 \tag{22}$$

where (a) comes from geometric series on the condition $R < 1$, i.e. $p > \xi$. After straightforward algebraic manipulation, the final expression at stable state is obtained:

$$\pi_0 = \frac{p-\xi}{p}, \forall\ p > \xi \tag{23}$$

Since π_0 is the probability to have an empty buffer, and hence an inactive BS, the activity probability of a BS is

$$q = 1 - \pi_0 = \frac{\xi}{p}, \forall\ p > \xi \tag{24}$$

If $R \geq 1$, then geometric series in (22) does not converge and the only solution to have (21) satisfied is that $\pi_i \to 0$, i.e. all states are transient and BS are always active, i.e. $q = 1$.

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