

Distributed Decisions on Optimal Load Balancing in Loss Networks

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Abstract—When multiple users share a common link in direct transmission, packet loss and link congestion may occur due to the simultaneous arrival of traffics at the source node. To tackle this problem, users may resort to an indirect path: the packet flows are first relayed through a sidelink to another source node, then transmitted to the destination. This behavior brings the problems of packet routing or load balancing: (1) how to maximize the total traffic in a collaborative way; (2) how self-interested users choose routing strategies to minimize their individual packet loss independently. In this work, we propose a generalized mathematical framework to tackle the packet and load balancing issue in loss networks. In centralized scenarios with a planner, we provide a polynomial-time algorithm to compute the system optimum point where the total traffic rate is maximized. Conversely, in decentralized settings with autonomous users making distributed decisions, the system converges to an equilibrium where no user can reduce their loss probability through unilateral deviation. We thereby provide a full characterization of Nash equilibrium and examine the efficiency loss stemming from selfish behaviors, both theoretically and empirically. In general, the performance degradation caused by selfish behaviors is not catastrophic; however, this gap is not monotonic and can have extreme values in certain specific scenarios.

Index Terms—load balancing, Nash equilibria, price of anarchy, network congestion, sidelink

I. INTRODUCTION

Since the seminal work of Erlang [1], loss networks have played a crucial role in analyzing and optimizing stochastic systems involving simultaneous resource utilization, and non-backlogging workloads (for an extensive overview, see [2]). Meanwhile, in the post-5G era, cloud-enabled networks have emerged as a dominant architecture, where multiple servers collect data from users and relay it to a central hub for final processing. To guarantee network efficacy, that is no server is either overburdened or underutilized, load balancing strategies are well studied, e.g., [3]. In this context, loss networks provide valuable mathematical frameworks for comprehending and enhancing load distribution within cloud-enabled networks.

Early load balancing research for cloud-enabled networks focused on centralized scenarios, where a centralized planner scheduled workloads to optimize aspects like performance-energy tradeoffs [3] and algorithmic considerations [4], [5]. However, due to the stringent latency requirement for real-time decisions and the increasing signaling overhead caused by the large-scale deployment of servers and massive users, distributed decisions become a better solution. In this context,

the complexity of the problem increases due to the non-cooperative and competitive behaviors among users within the system.

To address the challenges of load balancing in a distributed way, game theory provides a mathematical framework that describes and analyzes scenarios with interactive decisions [6]. Till now, some studies have demonstrated the efficacy of game-theoretic models in addressing load balancing problems. For instance, Mondal et al. [7] developed a game-theoretic model for load balancing among competitive cloudlets, while Yi et al. [8] investigated a similar problem, incorporating additional considerations of queue-aware strategies. In [9], [10], symmetric loss models where each source has an equal number of users are considered. However, previous studies mostly focused on limited cases of identical user strategies, which may not reflect real-world scenarios, i.e., different users may have different objectives and preferences. Therefore, further research is needed to develop game-theoretic models that can address the challenges of load balancing in a more general and realistic manner.

In this paper, we employ game theory to address load balancing in both distributed and centralized environments, where users have non-identical strategies and the number of users is not evenly distributed. Specifically, we consider the load balancing in a cloud-enabled network consisting of m source nodes (servers) $\{s_1, \dots, s_m\}$ and one destination node (central hub) d . Each source s_i has n_i users seeking service, and the traffic originating from each user is assumed to follow an independent Poisson point process with an identical rate. The nodes in the network are connected by two types of communication links, namely sidelinks that connect two sources, and direct links that connect a source and destination. The sidelink has a random identical independent distribution (i.i.d) loss with a fixed probability q , and the direct link has a congestion loss that depends on the arrival rate and service rate of each server.

The user cannot split its traffic, and has to determine how to route all of its traffic from the source node arrived at to the destination node. There are two approaches for the traffic transmission: a direct path (DP) in which the packet goes directly from the source arrived at to the destination, and an indirect path (IP) in which the packet is first relayed to another source node and then takes the direct link from that node to the destination. We treat packet loss probability as the performance

metric in load balancing, instead of additive costs like delay or fees in classical routing games [10], [11], resulting in a non-additive and non-convex optimization process. Each user aims to minimize its own loss probability and engage in a game by strategically selecting its own path. In the end, no user can reduce its loss probability by unilateral deviation and reaches the state of *Nash Equilibrium* (NE).

A. Our Contributions

Our work contributes to the load balancing game in the following aspects: First, we prove two lemmas related to the optimal solution when a centralized planner exists. Based on these lemmas, a low-complexity algorithm that maximizes the total traffic is proposed. Second, we study the decentralized environment where decisions are made by autonomous and self-interested users. The sufficient and necessary conditions on NE are derived, which depend on the number of users on direct path and each indirect path. Moreover, since a NE may be suboptimal, we use the price of anarchy (PoA) [12] to measure the gap between the NE led by users' selfish behaviors and the system optimum achieved by the centralized planner.

The rest of the paper is structured as follows. The formal model and notations are presented in Section II. In Section III, we provide details to compute the optimal solution that maximizes the total traffic when a centralized planner exists. In Section IV, we study the NE in the decentralized decision-making scenarios, and analyzed the efficiency loss stemming from selfish behaviors. In Section V, a fine-grained analysis is performed on the existence of NE in various network configurations for a specific scenario involving two source nodes. Numerical results are presented and discussed in Section VI. Finally, Section VII concludes the paper and outlines some future work.

B. Other related works

Routing games. As a special class of congestion games, routing games in a network are problems of routing traffic to achieve the best possible network performance, and have been studied within various contexts and within various communities, for example, the mathematics community [13], the telecommunications [14], and theoretical computer science [15], [16]. The above references have all in common a cost framework which is additive over links, such as delays or tolls, and is flow conserving (the amount entering a node equals the amount leaving it). Routing games with non-additive cost in loss networks are studied in [17].

Braess-like paradox in distributed systems. The Braess-like paradox is said to occur in a network system with distributed behaviors if adding an extra link or adding communication capacity to the system leads to a worse system performance. It widely exists in transportation networks and queuing networks. Bean et al. [18] show that it can occur in loss networks. Kameda et al. [19] consider a model similar to ours in that a job (packet) can be processed directly or indirectly; however, they do not consider the loss probability. They identify a Braess-like paradox in which adding capacity to the channel may

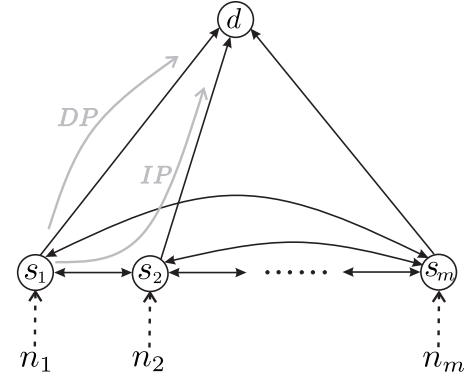


Fig. 1: An illustration of the game model.

degrade the system performance on the response time. Kameda and Pourtallier [20] characterize conditions under which such paradoxical behavior occurs, and give examples in which the degradation of performance may increase without bound.

II. MODEL AND PRELIMINARIES

We abstractly model our problem using a graph. Consider a network with m source nodes $S = \{s_1, \dots, s_m\}$ and one destination node d . For each source node $s_i \in S$, let N_i be the set of users arriving at s_i , and $n_i = |N_i|$ be the number of such users. Without loss of generality, we assume $n_1 > n_2 > \dots > n_m$. Denote $[m] = \{1, \dots, m\}$. There is a total of $n = \sum_{i \in [m]} n_i$ users in the system, who are self-interested players in the game. We say players and users interchangeably throughout this paper. Each user is identified with a flow (or traffic) of packets, which originates from the user and is assumed to form an independent Poisson process with an identical rate ϕ . See Fig. 1 for illustration.

Each user controls its route that all its packets should follow. For a user associated with $s_i \in S$, there are only two types of routes to ship these packets to the destination d : either a direct path (DP) (s_i, d) , or an indirect two-hop path (IP) (s_i, s_j, d) for some $s_j \neq s_i$, in which the packet is first sent to another source s_j by the side link (s_i, s_j) , and then passes through the direct link (s_j, d) .

Strategies. For every source s_i , each user $k \in N_i$ decides a one-shot strategy $\mathbf{p}_k^{(i)} = (p_{k1}^{(i)}, \dots, p_{km}^{(i)})^T \in [0, 1]^m$ with $\sum_{j \in [m]} p_{kj}^{(i)} = 1$, where $p_{ki}^{(i)}$ is the probability of routing all packets through DP, and $p_{kj}^{(i)}$ ($j \in [m], j \neq i$) is the probability of routing all packets through IP (s_i, s_j, d) . When no confusion arises, we simply write the strategy $\mathbf{p}_k^{(i)}$ as \mathbf{p}_k . We focus on *pure strategies* in this paper: a strategy \mathbf{p}_k is *pure* if $\|\mathbf{p}_k\|_\infty = 1$, i.e., user k deterministically selects a route with probability 1 (for example, $\mathbf{p}_k = (0, 1, 0, \dots, 0)^T$). Let $\mathbf{p} = (\mathbf{p}_1, \dots, \mathbf{p}_n)$ be the strategy profile of all users.

Loss probability and loss rate. There are two types of losses: (1) Losses on side links. We assume that a packet originating from node s_i and relayed to node s_j is lost with a fixed probability q for every side link (s_i, s_j) , independently of any other loss. Denote by $\bar{q} = 1 - q$ the probability that a packet

is successfully relayed. (2) Congestion losses on direct links. We assume that there is no buffer to restore the backlogged packets, so a packet will be lost when it enters the direct path which is occupied for the transmission of another packet. The transmission time of a packet on a direct link (s_i, d) is a random variable σ following a distribution \mathcal{X} , which is assumed to be an identically independent distribution (i.i.d) for all packets.

Given strategy profile \mathbf{p} , user $k \in N_i$ continuously sends packets that follow an independent Poisson process with rate $p_{ki}^{(i)} \cdot \phi$ to DP (s_i, d) , and an independent Poisson process of packets with rate $p_{kj}^{(i)} \cdot \phi$ to IP (s_i, s_j, d) , for any $s_j \neq s_i$. Since there is a random loss on the side link (s_i, s_j) , the flow of packets from user $k \in N_i$ that arrive at the node s_j is also a Poisson process with rate $\bar{q}p_{kj}^{(i)}\phi$.

Thus, for each source $s_i \in S$, the flow over the link (s_i, d) is Poisson distributed with a traffic rate $T_i(\mathbf{p})$ given by

$$T_i(\mathbf{p}) = \sum_{k \in N_i} p_{ki}^{(i)} \phi + \sum_{j \in [m] \setminus \{i\}} \sum_{k \in N_j} p_{kj}^{(j)} \bar{q} \phi. \quad (1)$$

When no confusion arises, we simply write $T_i(\mathbf{p})$ as T_i .

The probability of no congestion loss on the direct link (s_i, d) equals the probability that there is no arrival during a transmission time σ , which is given by $\mathbb{E}_{\sigma \sim \mathcal{X}} e^{-T_i \sigma}$. As usual practice, assume \mathcal{X} is an exponential distribution with a rate parameter μ (service rate) and mean $\frac{1}{\mu}$. Thus the probability of no congestion loss on (s_i, d) is

$$\mathbb{E}_{\sigma \sim \mathcal{X}} e^{-T_i \sigma} = \int_0^{+\infty} \mu e^{-\mu \sigma} e^{-T_i \sigma} d\sigma = \frac{\mu}{T_i + \mu}, \quad (2)$$

and the loss probability on link (s_i, d) is $\frac{T_i}{T_i + \mu}$.

Given the strategy profile \mathbf{p} , for $s_i \in S$ and $k \in N_i$, the loss rate of user k is defined as

$$LR_k(\mathbf{p}) = \left[p_{ki}^{(i)} \frac{T_i}{T_i + \mu} + (1 - p_{ki}^{(i)}) q + (1 - q) \sum_{j \in [m] \setminus \{i\}} p_{kj}^{(i)} \frac{T_j}{T_j + \mu} \right] \phi, \quad (3)$$

and the loss probability of user k is $\frac{LR_k(\mathbf{p})}{\phi}$.

Total traffic. Regarding the system efficiency, we measure it by the total traffic rate arriving at the destination d . Given the strategy profile \mathbf{p} , the total traffic rate $TR(\mathbf{p})$ of the system can be derived in two ways. The first expression is derived as the summation of successful transmission rates on direct links:

$$\begin{aligned} TR(\mathbf{p}) &= \sum_{i \in [m]} T_i \cdot \frac{\mu}{T_i + \mu} \\ &= \mu \left[m - \sum_{i \in [m]} \frac{\mu}{\sum_{k \in N_i} p_{ki}^{(i)} \phi + \sum_{j \in [m] \setminus \{i\}} \sum_{k \in N_j} p_{kj}^{(j)} \bar{q} \phi + \mu} \right] \end{aligned} \quad (4)$$

where T_i is the traffic rate over link (s_i, d) , and $\frac{\mu}{T_i + \mu}$ is the probability of no congestion loss on (s_i, d) .

The second expression is from users' perspective:

$$TR(\mathbf{p}) := \sum_{i \in [m]} \sum_{k \in N_i} (\phi - LR_k), \quad (5)$$

where $\phi - LR_k(\mathbf{p})$ is the traffic rate of user $k \in N_i$ that successfully arrive at d . It is not hard to see that (4) and (5) are equivalent.

Nash equilibria. A Nash equilibrium (NE) is a strategy profile where no player can decrease its loss probability by unilaterally deviating to any other strategy. Formally, we give a definition.

Definition 1. A strategy profile \mathbf{p} is a Nash equilibrium, if for any source $s_i \in S$ and any player $k \in N_i$, we have

$$LR_k(\mathbf{p}_k, \mathbf{p}_{-k}) \leq LR_k(\mathbf{p}'_k, \mathbf{p}_{-k}),$$

where \mathbf{p}'_k can be any feasible strategy of player k , and \mathbf{p}_{-k} is the strategy profile of all other players.

We measure the efficiency of NEs by the price of anarchy (PoA) [12], which is defined as the ratio between social efficiencies in an optimal solution and in the worst NE. Formally, given an instance Γ of this game, we define

$$PoA(\Gamma) = \frac{TR(opt)}{\min_{\mathbf{p} \in \mathbb{NE}} TR(\mathbf{p})},$$

where opt is an optimal solution of Γ , and \mathbb{NE} is the set of all NEs. The PoA of the whole game is defined as the maximum over all instances, that is, $PoA = \max_{\Gamma} PoA(\Gamma)$.

III. CENTRALIZED ANALYSIS

The main technical results of the paper are presented now. We show how to compute an optimal solution that maximizes the total traffic. Note that the total traffic rate depends on the number of users working on each source by DP or IP, but not the users' identity. Given a strategy profile \mathbf{p} , let $u_i = |\{k \in N_i \mid p_{ki}^{(i)} = 1\}|$ be the number of users working with DP (s_i, d) , and let $v_i = |\{k \in N_j, j \in [m] \setminus \{i\} \mid p_{ki}^{(j)} = 1\}|$ be the users working with IP through link (s_i, d) . Define $y_i = u_i + v_i$ as the number of users who choose source s_i (including both DP and IP).

Lemma 1. In any optimal solution, for any source s_i , either $u_i = n_i$ or $v_i = 0$ or both hold.

Proof. Let \mathbf{p} be an optimal solution. Suppose for contradiction that $u_i < n_i, v_i > 0$ for some source s_i . Then there exists a user (say, k) in N_i who chooses IP (say, $(s_i, s_{i'}, d)$ for some $i' \neq i$). Also, since $v_i > 0$, there exist a source $s_j \neq s_i$ and a user $l \in N_j$ who chooses IP (s_j, s_i, d) . The total traffic rate is $TR(\mathbf{p}) = \frac{\mu T_i}{T_i + \mu} + \sum_{w \in [m] \setminus \{i\}} \left(\frac{\mu T_w}{T_w + \mu} \right)$.

Now we show that the total traffic rate can be improved by revising \mathbf{p} . Let user $k \in N_i$ choose DP, and let user $l \in N_j$ choose IP $(s_j, s_{i'}, d)$. Fixing all others' strategies, denote the new strategy profile by \mathbf{p}' , and define u'_i, v'_i accordingly. Note that $u'_i = u_i + 1, v'_i = v_i - 1$, and $T_w(\mathbf{p}') = T_w(\mathbf{p})$ for all source $s_w \neq s_i$. Since $q > 0$, we have

$$T_i(\mathbf{p}') = (u_i + 1)\phi + (v_i - 1)\bar{q}\phi > u_i\phi + v_i\bar{q}\phi = T_i(\mathbf{p}).$$

So $TR(\mathbf{p}') > TR(\mathbf{p})$, contradicting to the optimality. \square

Lemma 1 indicates that if a source (say s_i) provides service to the users of other sources, then all users of s_i choose DP.

Lemma 2. *In any optimal solution, there must exist $\tilde{i} \in [m]$, such that $v_l = 0$ for all $l \leq \tilde{i}$, and $u_j = n_j$ for all $j > \tilde{i}$.*

Proof. Given an optimal solution \mathbf{p} , suppose for contradiction that there exist $i, j \in [m]$ ($i < j$) such that $v_i > 0$ and $u_j < n_j$. By Lemma 1, we have $u_i = n_i$ and $v_j = 0$. There exists a source $s_{i'}$ and a user $k \in N_{i'}$ selecting IP $(s_{i'}, s_i, d)$. There exists a source $s_{j'}$ ($j' \neq j$) and a user $k' \in N_{j'}$ selecting IP $(s_j, s_{j'}, d)$. Note that when $i' = j$ and $j' = i$, users k and k' may coincide. The total traffic rate is

$$\begin{aligned} TR(\mathbf{p}) &= \frac{\mu T_i}{T_i + \mu} + \frac{\mu T_j}{T_j + \mu} + \sum_{w \in [m] \setminus \{i, j\}} \left(\frac{\mu T_w}{T_w + \mu} \right) \\ &= \mu \left(2 - \frac{1}{T_i + \mu} - \frac{1}{T_j + \mu} \right) + \sum_{w \in [m] \setminus \{i, j\}} \left(\frac{\mu T_w}{T_w + \mu} \right). \end{aligned}$$

Now we show that the total traffic rate can be improved by revising \mathbf{p} . Let user k choose IP $(s_{i'}, s_{j'}, d)$ if $i' \neq j'$ and choose DP $(s_{i'}, d)$ if $i' = j'$. Let user $k' \in N_j$ choose DP. Fixing all others' strategies, denote the new strategy profile by \mathbf{p}' , and define u'_i, v'_i accordingly. Note that $v'_i = v_i - 1$, $u'_j = u_j + 1$, and $T_w(\mathbf{p}') = T_w(\mathbf{p})$ for all other sources $s_w \neq s_i, s_j$. Since $i < j$, it follows that $n_i \geq n_j > u_j$. Therefore, we have

$$\begin{aligned} \frac{1}{T_i + \mu} + \frac{1}{T_j + \mu} &= \frac{1}{n_i \phi + v_i \bar{q} \phi + \mu} + \frac{1}{u_j \phi + \mu} \\ &> \frac{1}{n_i \phi + v'_i \bar{q} \phi + \mu} + \frac{1}{u'_j \phi + \mu} = \frac{1}{T'_i + \mu} + \frac{1}{T'_j + \mu}, \end{aligned}$$

which indicates that $TR(\mathbf{p}) < TR(\mathbf{p}')$, a contradiction. \square

Lemma 2 shows that there exists a threshold \tilde{i} : 1) if $i > \tilde{i}$, all users from s_i chose DP; 2) if $i \leq \tilde{i}$, portion users chose DP, and portion users chose IP.

Now we are ready to present Algorithm 1. The main idea is searching for \tilde{i} in Lemma 2. For each candidate of \tilde{i} , let B be the number of users selecting IP, all of whom come from $L = \{s_l \mid l \leq \tilde{i}\}$, and go to $R = \{s_j \mid j > \tilde{i}\}$. For every possible value of B , we compute the best possible way for extracting the B users from L and distributing them over R .

In Algorithm 1, step (a) is to make T_l (and thus no congestion probability $\frac{\mu}{T_l + \mu}$) as equal as possible for $l \in [\tilde{i}]$. This can be realized by initializing $u_l = n_l$, and then removing players one by one from the highest u_l and updating until B players have been removed. The goal of step (b) is to make T_j (and thus $\frac{\mu}{T_j + \mu}$) as even as possible. This can be realized by initializing $v_j = 0$, then adding users one by one to $v_{j'} = \arg \min_{j > \tilde{i}} n_j \phi + v_j \bar{q} \phi + \mu$ and updating, until B players have been added. These two steps guarantee that the B loads are distributed in an optimal way to maximize the traffic rate.

Theorem 1. *Algorithm 1 returns an optimal solution for maximizing the total traffic, and runs in $O(mn^2)$ time.*

Proof. In the first loop, we traverse all indexes in $[m]$ to find the \tilde{i} in Lemma 2. In the second loop, we traverse all possible

ALGORITHM 1: Computing an optimal solution.

Input: m source with $n_1 \geq n_2 \geq \dots \geq n_m$, ϕ, μ, q

Output: $(u_i^*, v_i^*)_{i \in N}$

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1: Initialize  $u_i = v_i = 0$  for all  $i \in [m]$ .  $TR^* = 0$ 
2: for  $\tilde{i} = 1, \dots, m$  do
3:   Let  $v_l = 0$  for all  $l = 1, 2, \dots, \tilde{i}$ 
4:   Let  $u_j = n_j$  for all  $j = \tilde{i} + 1, \dots, m$ 
5:   for  $B = 1, 2, \dots, \sum_{l=1}^{\tilde{i}} n_l$  do
6:     (a) Compute  $(u_l)_{l \in [\tilde{i}]}$  s.t.  $\sum_{l=1}^{\tilde{i}} (n_l - u_l) = B$ 
        and the values of  $u_l$  are as equal as possible.
7:     (b) Compute  $(v_j)_{j > \tilde{i}}$  s.t.  $\sum_{j=\tilde{i}+1}^m v_j = B$  and the
        values of  $n_j \phi + v_j \bar{q} \phi + \mu$  are as equal as possible.
8:     (c) Compute  $TR$  with respect to  $(u_i, v_i)_{i \in N}$ 
9:     if  $TR > TR^*$  then
10:        $TR^* \leftarrow TR$ , and  $(u_i^*, v_i^*)_{i \in N} \leftarrow (u_i, v_i)_{i \in N}$ 
11:     end if
12:   end for
13: end for

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numbers of users who select IP, and given any such a number B , we extract the B users from $\{s_l \mid l \leq \tilde{i}\}$ and distribute them over $\{s_j \mid j > \tilde{i}\}$ in an optimal way to maximize the traffic rate. So all possible optimal solutions have been searched by the algorithm, giving the optimality. For the time complexity, we have m iterations in the first loop, at most n iterations in the second loop, and the time for each iteration is $O(n)$. \square

Intuitively, when the transmission loss probability is sufficiently large, all packets should go through DP; when there is no transmission loss, the load of packets should be distributed evenly over all sources. We verify the intuition as follows.

Corollary 1. *If $q = 1$, the unique optimal solution is that all users choose DP (i.e., $u_i = n_i, v_i = 0, \forall i \in M$). If $q = 0$, a strategy profile \mathbf{p} is optimal if and only if $|y_i - y_j| \leq 1$ for all $i, j \in [m]$.*

Proof. If $q = 1$, $TR = \sum_{i \in [m]} \frac{\mu u_i \phi}{u_i \phi + \mu}$ is increasing with respect to every u_i . By the monotonicity, the optimum is achieved when $u_i = n_i$. If $q = 0$, suppose for contradiction that there exist $i, j \in [m]$ in an optimal solution \mathbf{p} such that $y_i - y_j \geq 2$. The total traffic rate is $TR = \mu(m - \sum_{k \in [m]} \frac{\mu}{y_k \phi + \mu})$. Consider a new strategy profile \mathbf{p}' with $y'_i = y_i - 1$, $y'_j = y_j + 1$, i.e., a user who chooses source s_i deviates to s_j . Then the total traffic rate becomes $TR' = \mu(m - \sum_{k \in [m] \setminus \{i, j\}} \frac{\mu}{y_k \phi + \mu} - \frac{\mu}{y'_i \phi + \mu} + \frac{\mu}{y'_j \phi + \mu}) > TR$, a contradiction. \square

IV. DECENTRALIZED ANALYSIS

In this section, we study the Nash equilibria in the decentralized decision-making scenario where each user makes a decision on the choice of DP or IP.

A. Characterization of NEs

A NE should satisfy that: for a user selecting DP, its loss rate will not decrease if it deviates to any IP; for a user selecting

IP, its loss rate will not decrease if it deviates to DP or another IP. We formalize it as the following characterization.

Theorem 2. *Given an arbitrary strategy profile \mathbf{p} with $(u_i, v_i)_{i \in [m]}$, let $i^* \in \arg \min_{i \in [m]} \{u_i + v_i \bar{q}\}$, and let $x_{ij} \in \{0, 1\}$ be an indicator where $x_{ij} = 1$ if there exists at least one user selecting IP (s_i, s_j, d) . Then, \mathbf{p} is a NE, if and only if the following conditions are satisfied:*

(i) *for all $i \in [m]$ with $u_i > 0$, we have*

$$\bar{q}(u_i + v_i \bar{q}) \leq u_{i^*} + v_{i^*} \bar{q} + \bar{q} + \frac{q\mu}{\phi}; \quad (6)$$

(ii) *for all $i, l \in [m]$ with $x_{il} = 1$, we have*

$$u_l + v_l \bar{q} \leq \min \left\{ \bar{q}(u_i + 1 + v_i \bar{q}) - \frac{q\mu}{\phi}, u_{i^*} + v_{i^*} \bar{q} + \bar{q} \right\} \quad (7)$$

Proof. Suppose \mathbf{p} is a NE. Consider an arbitrary source $s_i \in S$ and arbitrary user $k \in N_i$.

Case 1. User k selects DP in \mathbf{p} (denoted as $\mathbf{p}_k^{(i)}$ where $p_{ki}^{(i)} = 1$). If it deviates to IP (s_i, s_j, d) where $j \neq i$ (denoted as $\mathbf{p}'_k^{(i)}$ where $p_{kj}^{(i)} = 1$), by Definition 1, we have $LR_k(\mathbf{p}_k^{(i)}, \mathbf{p}_{-k}) \leq LR_k(\mathbf{p}'_k^{(i)}, \mathbf{p}_{-k})$. It is equivalent to

$$\begin{aligned} 1 - \frac{\mu}{u_i \phi + v_i \bar{q} \phi + \mu} &\leq q + \bar{q} \cdot \left(1 - \frac{\mu}{u_j \phi + (v_j + 1) \bar{q} \phi + \mu} \right) \\ \Leftrightarrow \frac{\bar{q}}{u_j \phi + (v_j + 1) \bar{q} \phi + \mu} &\leq \frac{1}{u_i \phi + v_i \bar{q} \phi + \mu} \\ \Leftrightarrow \bar{q}(u_i + v_i \bar{q}) - \frac{q\mu}{\phi} &\leq u_j + (v_j + 1) \bar{q}, \end{aligned}$$

The above inequality should hold for all $j \neq i$, and thus is equivalent to Equation (6).

Case 2. User k selects IP (s_i, s_l, d) in \mathbf{p} . If it deviates to DP (s_i, d) , by Definition 1, we should have $LR_k(\mathbf{p}_k^{(i)}, \mathbf{p}_{-k}) \leq LR_k(\mathbf{p}'_k^{(i)}, \mathbf{p}_{-k})$. It is equivalent to

$$\begin{aligned} q + \bar{q} \cdot \left(1 - \frac{\mu}{u_l \phi + v_l \bar{q} \phi + \mu} \right) &\leq 1 - \frac{\mu}{(u_i + 1) \phi + v_i \bar{q} \phi + \mu} \\ \Leftrightarrow \frac{1}{(u_i + 1) \phi + v_i \bar{q} \phi + \mu} &\leq \frac{\bar{q}}{u_l \phi + v_l \bar{q} \phi + \mu} \\ \Leftrightarrow u_l + v_l \bar{q} &\leq \bar{q}(u_i + 1 + v_i \bar{q}) - \frac{q\mu}{\phi}. \end{aligned}$$

Moreover, a NE must guarantee that user k will not deviate to another IP $(s_i, s_j, 0)$, and thus we should have

$$\begin{aligned} 1 - \frac{\mu}{u_l \phi + v_l \bar{q} \phi + \mu} &\leq 1 - \frac{\mu}{u_j \phi + (v_j + 1) \bar{q} \phi + \mu} \\ \Leftrightarrow u_l + v_l \bar{q} &\leq u_j + (v_j + 1) \bar{q}. \end{aligned}$$

Note that the above inequality should hold for all $j \neq i, l$. Therefore, we obtain Equation (7). \square

B. Price of Anarchy

We investigate the price of anarchy in this section, which measures the efficiency of NE. We give an upper bound on the optimal total traffic rate, and a lower bound on the total traffic rate of any NE.

Lemma 3. *In an optimal solution \mathbf{p} , the total traffic rate is $TR(\mathbf{p}) \leq \mu m \left(1 - \frac{\mu}{n_1 \phi + \mu} \right)$.*

Proof. Let i be the index stated in Lemma 2. It suffices to show with the proof by contradiction that in the optimal solution \mathbf{p} , $u_i + v_i \bar{q} \leq n_i$. First, for $i = 1$, we have $v_1 = 0$, and thus it satisfies $u_1 + v_1 \bar{q} = u_1 \leq n_1$. For any $i > 1$, suppose for contradiction that $u_i + v_i \bar{q} > n_i$. Then $v_i > 0$, and there exists a source s_j and a user $k \in N_j$ that chooses the IP (s_j, s_i, d) , i.e., $u_j < n_j$. By Lemma 1, it must be $v_j = 0$, and thus $T_j = u_j \phi$. Denote the strategy as \mathbf{p} with $p_{ki}^{(j)} = 1$. The total traffic rate is

$$TR(\mathbf{p}) = \mu \left(m - \frac{\mu}{T_j + \mu} - \frac{\mu}{T_i + \mu} - \sum_{w \in [m] \setminus \{j\}} \frac{\mu}{T_w + \mu} \right) \quad (8)$$

We show that the total traffic rate can be improved with user $k \in N_j$ deviating from IP (s_j, s_i, d) to DP (s_j, d) . Fixing the strategies of all others, denote by \mathbf{p}' the new strategy profile, and define $(u'_w, v'_w, T'_w)_{w \in [m]}$ accordingly. Note that $u'_j = u_j + 1$, $v'_i = v_i - 1$, $u'_i = u_i$, and $T'_w = T_w$ for any $w \in [m] \setminus \{j\}$. Since $u_i + v_i \bar{q} > n_1 \geq n_j \geq u_j$, we have

$$\begin{aligned} \frac{1}{T_j + \mu} + \frac{1}{T_i + \mu} &= \frac{1}{u_j \phi + \mu} + \frac{1}{u_i \phi + v_i \bar{q} \phi + \mu} \\ &> \frac{1}{u'_j \phi + \mu} + \frac{1}{u'_i \phi + v'_i \bar{q} \phi + \mu} = \frac{1}{T'_j + \mu} + \frac{1}{T'_i + \mu}. \end{aligned}$$

It indicates that $TR(\mathbf{p}') > TR(\mathbf{p})$ is a contradiction. Consequently, $u_i + v_i \bar{q} \leq n_i \leq n_1$. According (4), we have $TR(\mathbf{p}) \leq \mu m \left(1 - \frac{\mu}{n_1 \phi + \mu} \right)$. \square

Lemma 4. *Let $z = \min\{n_m, \frac{n}{4m} - \bar{q} - \frac{q\mu}{\phi}\}$. For every NE \mathbf{p} , the total traffic rate satisfies $TR(\mathbf{p}) \geq \mu(m - \frac{m\mu}{z\phi + \mu})$.*

Proof. Let $i^* = \arg \min_{i \in [m]} \{u_i + v_i \bar{q}\}$. Since $TR(\mathbf{p}) \geq \mu m \left(1 - \frac{\mu}{(u_{i^*} + v_{i^*} \bar{q}) \phi + \mu} \right)$, it suffices to prove that $u_{i^*} + v_{i^*} \bar{q} \geq z$. If $u_{i^*} + v_{i^*} \bar{q} \geq n_m$, it is done. We only need to consider the case when $u_{i^*} + v_{i^*} \bar{q} < n_m \leq n_{i^*}$. There exists some users in N_{i^*} selecting IP. By Equation (7), we have

$$u_{i^*} + v_{i^*} \bar{q} \leq \bar{q}(u_{i^*} + 1 + v_{i^*} \bar{q}) - \frac{q\mu}{\phi}.$$

By Theorem 2, for each $i \in [m]$, if $u_i > 0$, then $\bar{q}(u_i + v_i \bar{q}) \leq u_{i^*} + v_{i^*} \bar{q} + \bar{q} + \frac{q\mu}{\phi}$; if $v_i > 0$, then $u_i + v_i \bar{q} \leq u_{i^*} + v_{i^*} \bar{q} + \bar{q}$. In both cases, we obtain $u_i + v_i/2 \leq 2(u_{i^*} + v_{i^*} \bar{q} + \bar{q} + \frac{q\mu}{\phi})$. Summing up over all $i \in [m]$, we have

$$n/2 \leq \sum_{i \in [m]} (u_i + v_i/2) \leq 2m(u_{i^*} + v_{i^*} \bar{q} + \bar{q} + \frac{q\mu}{\phi}),$$

which implies that $u_{i^*} + v_{i^*} \bar{q} \geq \frac{n}{4m} - \bar{q} - \frac{q\mu}{\phi}$. \square

Theorem 3. *For any instance with m sources, the price of anarchy is $PoA \leq 1 + \frac{n_1 \mu}{n_1 z \phi + z \mu}$, where $z = \min\{n_m, \frac{n}{4m} - \bar{q} - \frac{q\mu}{\phi}\}$.*

Proof. Combining the upper bound in Lemma 3 and the lower bound on $TR(\mathbf{p})$ for any NE \mathbf{p} in Lemma 4, it follows

$$PoA \leq \frac{m - \frac{m\mu}{n_1\phi + \mu}}{m - \frac{m\mu}{z\phi + \mu}} = \frac{n_1(z\phi + \mu)}{z(n_1\phi + \mu)} = 1 + \frac{n_1\mu}{n_1z\phi + z\mu}.$$

□

V. A PARTICULAR CASE: TWO SOURCES

In this section, we focus on the special case of $m = 2$. That is, there are only two sources s_1 and s_2 . Assume w.l.o.g. that $n_1 \geq n_2$. For each user $k \in N_i$, there is only one IP. Accordingly, its strategy becomes $\mathbf{p}_k^{(i)} = (p_{k1}^{(i)}, p_{k2}^{(i)})$, $i = 1, 2$. And we have

$$n_1 = u_1 + v_2; \quad (9)$$

$$n_2 = u_2 + v_1. \quad (10)$$

The traffic rate $T_i(\mathbf{p})$ in (1) is rephrased as

$$T_i(\mathbf{p}) = \sum_{k \in N_i} p_{ki}^{(i)} \phi + \sum_{k \in N_j, j \neq i} p_{ki}^{(j)} \bar{q} \phi = u_i \phi + v_i \bar{q} \phi. \quad (11)$$

Given strategy profile \mathbf{p} , the set N is further partitioned into 4 subsets (V_1, V_2, V_3, V_4) where $V_1 = \{k \in N_1 \mid \mathbf{p}_k^{(1)} = (1, 0)\}$, $V_2 = \{k \in N_1 \mid \mathbf{p}_k^{(1)} = (0, 1)\}$, $V_3 = \{k \in N_2 \mid \mathbf{p}_k^{(2)} = (0, 1)\}$ and $V_4 = \{k \in N_2 \mid \mathbf{p}_k^{(2)} = (1, 0)\}$. Clearly, users in V_1 and V_3 choose DP, and users in V_2 and V_4 choose IP.

Suppose \mathbf{p} is a NE. We study the deviation of users in V_1, V_2, V_3, V_4 , respectively. For user $k \in V_1$, the strategy is $\mathbf{p}_k^{(1)} = (p_{k1}^{(1)}, p_{k2}^{(1)}) = (1, 0)$, and the loss rate in (3) is

$$LR_k(\mathbf{p}) = \frac{\phi T_1(\mathbf{p})}{T_1(\mathbf{p}) + \mu} = \left[1 - \frac{\mu/\phi}{u_1 + v_1 \bar{q} + \mu/\phi}\right] \phi.$$

When user $k \in V_1$ deviates to IP, the strategy profile becomes $\mathbf{p}' = (\mathbf{p}'_k, \mathbf{p}_{-k})$ where $\mathbf{p}'_k = (0, 1)$. The loss rate of user k becomes

$$LR_k(\mathbf{p}') = q\phi + \bar{q}\phi \frac{T_2(\mathbf{p}')}{T_2(\mathbf{p}') + \mu} = \left[1 - \frac{\bar{q}\mu/\phi}{u_2 + (v_2 + 1)\bar{q} + \mu/\phi}\right] \phi.$$

Since \mathbf{p} is NE, k has no incentive to deviate, and thus $LR_k(\mathbf{p}) \leq LR_k(\mathbf{p}')$, which is equivalent to

$$t_1(u_2) := \frac{q\mu/\phi + u_2(1 + \bar{q}^2) + (n_1 + 1)\bar{q} - n_2\bar{q}^2}{2\bar{q}} \geq u_1, \quad (12)$$

where $t_1(u_2)$ is a function with respect to variable u_2 .

For user $k \in V_2$ with strategy $\mathbf{p}_k^{(1)} = (0, 1)$, the loss rate is

$$LR_k(\mathbf{p}) = q\phi + \bar{q}\phi \frac{T_2(\mathbf{p})}{T_2(\mathbf{p}) + \mu} = \left[1 - \frac{\bar{q}\mu/\phi}{u_2 + v_2\bar{q} + \mu/\phi}\right] \phi.$$

When user $k \in V_2$ deviates to DP, the strategy profile becomes $\mathbf{p}' = (\mathbf{p}'_k, \mathbf{p}_{-k})$ where $\mathbf{p}'_k = (1, 0)$. The loss rate of k becomes

$$LR_k(\mathbf{p}') = \frac{\phi T_1(\mathbf{p}')}{T_1(\mathbf{p}') + \mu} = \left[1 - \frac{\mu/\phi}{u_1 + 1 + v_1\bar{q} + \mu/\phi}\right] \phi.$$

Since \mathbf{p} is NE, we have $LR_k(\mathbf{p}) \leq LR_k(\mathbf{p}')$, that is,

$$u_1 \geq \frac{q\mu/\phi + u_2(1 + \bar{q}^2) + (n_1 - 1)\bar{q} - n_2\bar{q}^2}{2\bar{q}} = t_1(u_2) - 1. \quad (13)$$

Symmetrically, for each user $k \in V_3$ and $k \in V_4$, since \mathbf{p} is NE, we have

$$t_2(u_1) - 1 \leq u_2 \leq t_2(u_1), \quad (14)$$

where

$$t_2(u_1) := \frac{q\mu/\phi + u_1(1 + \bar{q}^2) + (n_2 + 1)\bar{q} - n_1\bar{q}^2}{2\bar{q}}.$$

Note that Eq. (12) - (14) are the sufficient and necessary conditions for an arbitrary strategy \mathbf{p} to achieve NE. Now we are ready to give a characterization of NEs.

Theorem 4. Let \mathbf{p} be an arbitrary strategy profile for the game with two sources. Let u_1 and u_2 be the number of users in N_1 and N_2 who choose DP under \mathbf{p} , respectively. We have

- [1] when (a) $u_1 = n_1, u_2 < n_2$, or (b) $u_1 = 0, u_2 > 0$, \mathbf{p} cannot be a NE;
- [2] when $u_1 \in [0, n_1], u_2 \in [0, n_2]$, \mathbf{p} is NE if and only if $u_1 \geq t_1(u_2) - 1$ and $u_2 \geq t_2(u_1) - 1$;
- [3] when $u_1 \in (0, n_1), u_2 = n_2$, \mathbf{p} is NE if and only if $u_1 \in [t_1(u_2) - 1, t_1(u_2)]$;
- [4] when $u_1 = n_1, u_2 = n_2$, \mathbf{p} is NE if and only if $n_1\bar{q} \leq q\mu/\phi + n_2 + \bar{q}$.

Proof. Given \mathbf{p} , let (V_1, V_2, V_3, V_4) be a partition of N as defined above. We discuss the four cases.

Case 1. When (a) $u_1 = n_1$ and $u_2 < n_2$, V_4 is nonempty. If \mathbf{p} is a NE, it must satisfy $t_2(u_1) - 1 \leq u_2$. However, because $\bar{q}u_2 \leq n_1, \bar{q}u_2 \leq (n_2 - 1)\bar{q}$ and $q\mu/\phi > 0$, it cannot hold.

When (b) $u_1 = 0$ and $u_2 > 0$, V_2 is nonempty. If \mathbf{p} is a NE, it must satisfy Eq. (13), that is, $u_1 \geq t_1(u_2) - 1$. It follows that $0 = 2\bar{q}u_1 \geq q\mu/\phi + u_2(1 + \bar{q}^2) + (n_1 - 1)\bar{q} - n_2\bar{q}^2 \geq q\mu/\phi + 1 + (n_1 - 1)\bar{q} - (n_2 - 1)\bar{q}^2 \geq q\mu/\phi + 1 > 0$, a contradiction.

Case 2. When $u_1 \in [0, n_1], u_2 \in [0, n_2]$, V_2 and V_4 are nonempty. It is easy to see that \mathbf{p} is NE if and only if $u_1 \geq t_1(u_2) - 1$ and $u_2 \geq t_2(u_1) - 1$ are satisfied simultaneously.

Case 3. When $u_2 = n_2, u_1 \in (0, n_1)$, V_1, V_2, V_3 are nonempty, and V_4 is empty. \mathbf{p} is NE if and only if $t_1(u_2) - 1 \leq u_1 \leq t_1(u_2)$ and $u_2 \leq t_2(u_1)$ hold simultaneously. Moreover, note that $u_2 \leq t_2(u_1)$ is implied by $u_1 \geq t_1(u_2) - 1$. Therefore, the sufficient and necessary condition for NE is $u_1 \in [t_1(u_2) - 1, t_1(u_2)]$.

Case 4. When $u_1 = n_1, u_2 = n_2$, V_1, V_3 are nonempty, and V_2, V_4 are empty. \mathbf{p} is NE if and only if $u_1 \leq t_1(u_2)$ and $u_2 \leq t_2(u_1)$. It is easy to see that, it is equivalent to $n_1\bar{q} \leq q\mu/\phi + n_2 + \bar{q}$.

Note that every situation of u_1, u_2 is included in the above four cases. So we complete a characterization. □

Case 4 can be intuitively explained by considering the sidelink loss probability q over link (s_1, s_2) . If q is sufficiently high, no user would prefer the indirect path, and selecting the direct path would be a NE for all users. Conversely, when there is no transmission loss over sidelink (s_1, s_2) (i.e., $q = 0$), every user would prefer to use the source with fewer users.

Therefore, the profile of all users selecting DP is a NE only if the user distribution between the two sources is as even as possible, with $n_1 \leq n_2 + 1$. Based on Theorem 4, we give some interesting conclusions.

Corollary 2. *If a strategy profile with $u_1 = n_1, u_2 = n_2$ is optimal, then it is also a NE.*

Corollary 3. *A strategy profile with $u_1 = 0, u_2 = 0$ is a NE, if and only if (a) $n_1 = n_2 + 1, \bar{q} = 1$, or (b) $n_1 = n_2, n_1(1 - \bar{q}^2) \leq \bar{q} - \frac{q\mu}{\phi}$.*

Note that $u_1 \geq t_1(u_2) - 1$ and $u_2 \geq t_2(u_1) - 1$ cannot hold simultaneously when $q > \frac{2}{n}$, and $u_1 \geq t_1(n_2) - 1$ cannot hold when $n_1\bar{q} < q\mu/\phi + n_2 + \bar{q}$.

Corollary 4. *When $n_1\bar{q} < q\mu/\phi + n_2 + \bar{q}$ and $q > \frac{2}{n}$, the unique NE is that all users choose DP, i.e., $u_1 = n_1, u_2 = n_2$.*

We end this section by proving the existence of NE.

Theorem 5. *For any game instance with two sources, there exists a NE with $u_1 > 0$ and $u_2 = n_2$.*

Proof. By Theorem 4 (4), if $n_1\bar{q} \leq q\mu/\phi + n_2 + \bar{q}$, then the strategy profile that all users choose DP (i.e., $u_1 = n_1, u_2 = n_2$) is a NE. Otherwise, $n_1\bar{q} > q\mu/\phi + n_2 + \bar{q}$. Let \tilde{m} be an integer in interval $[\frac{q\mu/\phi + n_2 + n_1\bar{q} - \bar{q}}{2\bar{q}}, \frac{q\mu/\phi + n_2 + n_1\bar{q} + \bar{q}}{2\bar{q}}] = [t_1(n_2) - 1, t_1(n_2)]$, which always admits at least one integer. Note that $n_1 > \frac{q\mu/\phi + n_2 + n_1\bar{q} + \bar{q}}{2\bar{q}} \geq \tilde{m} > 0$. By Theorem 4, a strategy profile with $u_1 = \tilde{m}$ and $u_2 = n_2$ is a NE. \square

VI. NUMERICAL EXPERIMENTS

Through numerical simulations, we explore the impact of traffic condition on network performance, i.e., the total traffic rate and PoA. Recall that the traffic flow originating from each user is Poisson with rate ϕ , the service rate of each direct link is μ , and the loss probability over each side link is q . Assume $\phi = 1$ for normalization.

We first present the simulation results in two-source networks. In Fig. 2, the PoA and the total traffics are plotted under different q, μ and n_1 , showing a PoA of less than 1.08. Such a little gap between the optimal solution and the worst NE suggests that the gain of centralized-decision making over decentralized-decision making is trivial most of the time. As shown in Fig. 2a, the total traffic decreases with the increase of q , i.e., the increased loss rate on sidelink. On the other hand, the PoA is first increasing from 1 at $q = 0$, implying that the NE and optimal solution are the same with $u_1 = n_2 + v_1$. That is, we have the equal number of users on (s_1, d) and (s_2, d) in terms of both IP and DP. With the increase of q , the benefit of centralized-decision making is gradually unveiled. However, when p reaches a certain value, the PoA goes down to 1 quickly. An intuitive explanation is that, when q becomes larger than the loss probability on DP, no users will choose IP in NE. And this strategy is optimal as well.

In Fig. 2b, the traffic rates grow with the increase of μ due to the increased probability of no congestion in (2). This is, a high service rate help clear the collision and relieve

congestion on both DP and IP. The PoA curve indicates that either in overloaded or less congested scenarios, there is little improvement of centralized-decision making. In Fig. 2c, the increased number of users leads to an increase of traffic rate in spite of the rise in loss rate. What is more, PoA tends towards 1 for small and large n_1 . As the strategies in *opt* and NE are much similar for users at source s_1 , i.e., DP in less biased scenario and IP severely biased scenario.

In the multi-source network, while the optimal solution can be easily computed by Algorithm 1, it is difficult to find all NEs even given Theorem 2. Hence, we merely consider a small value of m and n (i.e., $m = 3$). The service rate and traffic arrival rate are fixed as $\mu = 1, \phi = 1$. Results are given in Fig. 3, which shows similar results in Fig. 2c. It is obvious that the growth of the total traffic slows down gradually, because given the service rate, an increase of n_1 aggravates the network congestion. Second, the increase of loss rate on sidelink, leads to the increase of loss rate on IP. As a result, more users choose DP instead, which in turn worsens the network congestion.

Figure 4 plots the performances for a range of q . When $q = 0$ and $q = 1$, the PoA is exactly 1. The PoA converges to 1 when q goes to 1, because when the service rate is large enough compared with arrival rate, there is a sufficiently small congestion loss and all users like to choose DP.

VII. CONCLUSION

In this work, we give a theoretical analysis of a load balancing game in cloud-enabled networks, in which the users want to minimize the loss probability of their packets with suitable routing strategies. In the centralized analysis, an efficient algorithm for maximizing the total traffic rate is proposed, according to Lemma 1 and Lemma 2. In the decentralized analysis, a characterization of Nash equilibrium is given, and the PoA is investigated. Numerical experiments show that the efficiency loss due to selfish behaviors is relatively small in most cases.

There are many future directions that are worth exploring. First, we only focus on pure strategies of players in this work, and an immediate and natural question is how the users act when mixed strategies are allowed. Second, it would be interesting to investigate heterogeneous servers (source nodes) where each s_i serves a different purpose or has a different service rate μ_i . Moreover, while we only consider direct path and one-hop indirect paths, a more general scenario where players can choose multi-hop indirect paths to the destination can be taken into consideration.

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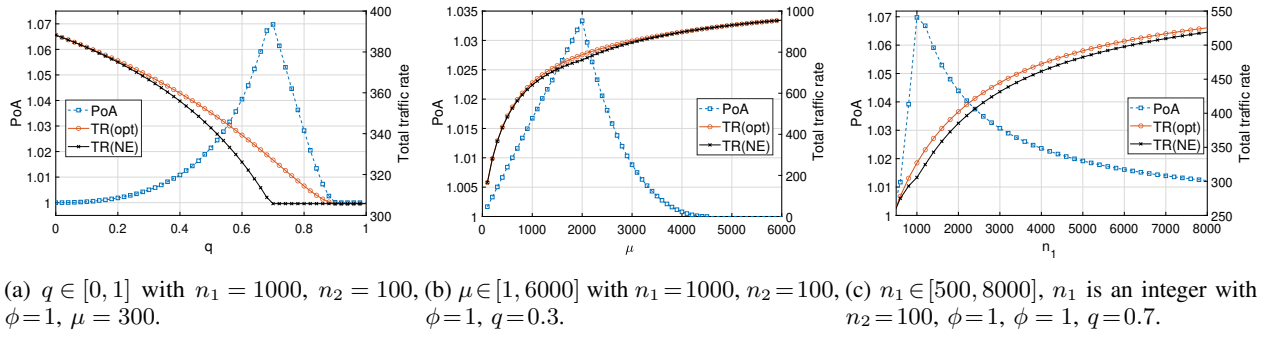


Fig. 2: The PoA and total traffics in *opt* and NE in two-source network

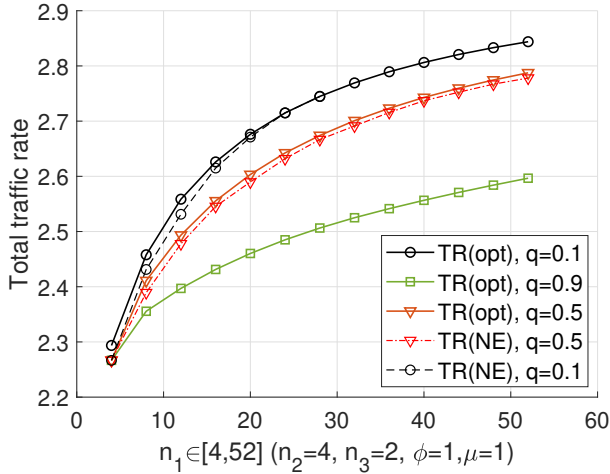


Fig. 3: The impact of n_1 on total traffics in multi-source network.

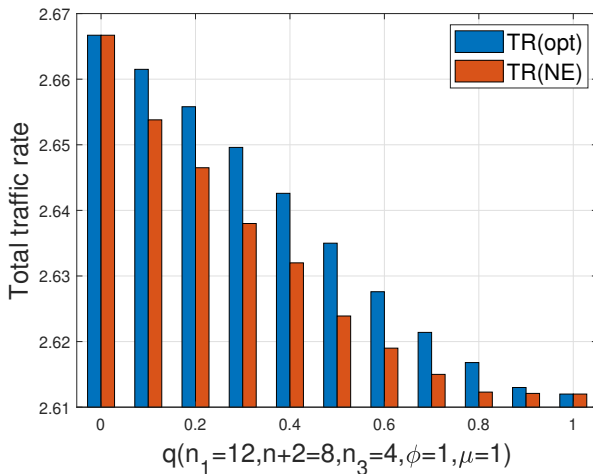


Fig. 4: Comparison of NE strategy and optimal strategy in terms of traffic rate.

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